IEEE Control Systems Magazine warmly welcomes your letters on any aspect of this magazine or control technology. Letters may be edited in consultation with the author. Please send all letters by post or e-mail to the editor-in-chief.

MISSING LINKS

I am contacting you concerning the article [1], which appeared in the December 2007 issue of *IEEE Control Systems Magazine*, coauthored by us and R. Venugopal. In this regard, I must bring to your attention the fact that I have discovered two additional articles on the subject. These articles present further insights into the properties of dimensions. The purpose of this letter is to briefly describe these insights.

The articles I am referring to are [2] and [3], which build on the book [4]. These works develop *orientation analysis*, which augments standard dimensional analysis in verifying candidate relationships and uncovering new relationships. Orientation analysis keeps track of the orientation of each quantity and ensures that, in each physical relationship, the quantities on both sides have the same orientation. Although this idea is intuitively clear, it has been widely ignored in textbooks.

From an orientation point of view, quantities such as length, velocity, acceleration, force, and moment have a direction in space, whereas quantities such as mass, time, energy, and power do not. Not surprisingly, the orientation of a position, velocity, or acceleration vector along the x-direction is different from that of a corresponding vector along the y-direction. From this point of view, two force vectors acting along different lines of action, although representing the same physical quantity, are fundamentally different because they have different directions. Although closely related, orientation is *not* the same as vector direction. For instance, area has orientation but is not a vector.

As an application of orientation analysis, consider the problem discussed in [2]–[4] of deriving an expression for the range R of a projectile fired horizontally from a height h, with a velocity v_0. Since $[h] = \mathrm{m}$, $[v_0] = \mathrm{m/s}$, $[R] = \mathrm{m}$, and $[g] = \mathrm{m/s}^2$, the problem involves four quantities and two dimensions. Therefore, it follows from the Buckingham Pi theorem given in [1] that there exist two dimensionless quantities, namely, $\Pi_1 = R/h$ and $\Pi_2 = v_0/\sqrt{hg}$. This result is not useful, however, for characterizing R in terms of v_0, h, and g. However, if we recognize that lengths along the vertical and horizontal directions have different orientations, and thus represent different physical dimensions, namely, m_x and m_y, we have $[h] = \mathrm{m}_y$, $[v_0] = \mathrm{m}_x/\mathrm{s}$, $[R] = \mathrm{m}_x$, and $[g] = \mathrm{m}_y/\mathrm{s}^2$. We then have four quantities and three dimensions, which yields the dimensionless quantity

$$\Pi_1 = \frac{v_0}{R}\sqrt{\frac{h}{g}},$$

and thus the useful expression $R = \Pi_1 v_0 \sqrt{h/g}$.

In addition to providing a tool for deriving and checking physical relationships, orientation analysis can be used to explain the nature of angles and dimensionless units such as radians. For example, an angle can be viewed as the ratio of two lengths along perpendicular directions (that is, radial and circumferential within the context of a circle), and thus angle has orientation, namely, along the direction perpendicular to the plane containing the two lengths, as noted in [2]. This point is consistent with the treatment of angle as a vector pointing out of the plane that contains the angle; this vector can be dotted with a moment vector to determine the work done by the moment vector when moving through an angle. Note that a ratio of identical dimensions, such as length/length, is dimensionless but can possess orientation.

Orientation analysis helps explain why radians are different from strain, which is also a ratio of lengths but does not have orientation. This distinction is not due to the fact that radians have a natural circle scale, namely, 2π, but rather the fact that strain is the ratio of lengths along the same direction. On the other hand, Poisson's ratio, which measures the ratio of the resulting bulge or shrinkage due to the strain in an orthogonal direction, *is* appropriately measured in radians. As another example, the distinguishing feature between moment and energy—which we explain in our article as being due to the distinction between a scalar and a vector—is that moment has orientation, whereas energy does not.

The rules and consequences of orientation analysis, which are explained in detail in [2] and [3], can be used to compute the orientations

Digital Object Identifier 10.1109/MCS.2008.920947

of various derived quantities. For example, as noted above, area has orientation, although volume is orientationless. Even more surprising is the consequence that $\cos\theta$ is orientationless, but $\sin\theta$ has orientation. Since the square of every orientation is orientationless, the identity $\cos^2\theta + \sin^2\theta = 1$ is orientationless. Along the same lines, $e^{\jmath\omega t} = \cos(\omega t) + \jmath\sin(\omega t)$ (where $\jmath \triangleq \sqrt{-1}$) is orientationless, which in turn implies that $\jmath$ has orientation. Since $\jmath$ represents a rotation about the z-axis, this observation seems reasonable. Although these observations are surprising, their consistency strikes me as beyond coincidental. Something deep and fundamental seems to be going on here.

Although orientation analysis offers insight into the nature of dimensionless units such as radians, there remain unanswered questions in the world of dimensions and units. For example, it occurs to me that the angle between a force vector and a displacement vector, whose dot product arises in computing work (that is, energy transferred), should not be measured in radians but rather in forcians. In other words, it seems that not all angles are created equal. Another mystery is the fact that orientation analysis does not offer a satisfactory explanation for the appearance of radians in the expression for natural frequency $\omega = \sqrt{k/m}$, where m is mass and k is the ratio of force to displacement. While we usually explain radians in terms of a ratio of lengths, the force and displacement in the simple harmonic oscillator are parallel, and thus orientation analysis suggests that the correct unit for natural frequency is not radian in the sense of orientation along the z-axis but rather the orientationless lengthian.

I suspect we haven't heard the last word on this subject. It often seems that definitively resolving fundamental issues is not a simple task.

Harish J. Palanthandalam-Madapusi
Syracuse University

References

[1] H.J. Palanthandalam-Madapusi, D.S. Bernstein, and R. Venugopal, "Dimensional analysis of matrices: State-space models and dimensionless units," *IEEE Contr. Sys. Mag.*, vol. 27, no. 6, pp. 100–109, Dec. 2007.

[2] D. Siano, "Orientational analysis—A supplement to dimensional analysis I," *J. Franklin Inst.*, vol. 320, pp. 267–283, 1985.

[3] D. Siano, "Orientational analysis, tensor analysis and the group properties of the SI Supplementary Units II," *J. Franklin Inst.*, vol. 320, pp. 285–302, 1985.

[4] H.E. Huntley, *Dimensional Analysis*. London: McDonald, 1952; reprinted by Dover, 1967.

Editor's Note: Perhaps a rationale for orientational analysis can be found in geometric algebra, where area is treated as a bivector. See C. Doran and A. Lasenby, *Geometric Algebra for Physicists*, Cambridge University Press, 2005.